Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced/Advanced Subsidiary

Wednesday 18 June 2014 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 6 questions in this question paper. The total mark for this paper is 75.
There are 20 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P43161A

1. A small bead is threaded on a smooth, straight horizontal wire which passes through the point $A(-3, 1)$ and the point $B(2, 5)$ in the x-y plane. The bead moves under the action of a horizontal force $\mathbf{F}$ of magnitude 8.5 N whose line of action is parallel to the line with equation $15x - 8y + 4 = 0$. The unit on both the x and y axes has length one metre. Find the work done by $\mathbf{F}$ as it moves the bead from A to B.

(8)

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2. A particle P moves in a plane so that its position vector, $\mathbf{r}$ metres at time t seconds, satisfies the differential equation

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \mathbf{r} = t\mathbf{i} + \mathrm{e}^{-t}\mathbf{j}$$

When $t = 0$ the particle is at the point with position vector $(\mathbf{i} + \mathbf{j})$ m.

Find $\mathbf{r}$ in terms of t.

(9)

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3. Three forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a rigid body at the points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ respectively.

$\mathbf{F}_1 = (2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ N and $\mathbf{r}_1 = (\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ m,
$\mathbf{F}_2 = (\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})$ N and $\mathbf{r}_2 = (3\mathbf{i} - \mathbf{j} - \mathbf{k})$ m,
$\mathbf{F}_3 = (-3\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ N and $\mathbf{r}_3 = (\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ m.

Show that the system is equivalent to a couple and find the magnitude of the vector moment of this couple.

(9)

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4. A spacecraft is travelling in a straight line in deep space where all external forces can be assumed to be negligible. The spacecraft decelerates by ejecting fuel at a constant speed k relative to the spacecraft, in the direction of motion of the spacecraft. At time t, the spacecraft has speed v and mass m.

(*a*) Show, from first principles, that while the spacecraft is ejecting fuel,

$$\frac{\mathrm{d}v}{\mathrm{d}m} - \frac{k}{m} = 0$$

(5)

At time $t = 0$, the spacecraft has speed U and mass M.

(*b*) Find the mass of the spacecraft when it comes to rest.

(6)

Given that $m = M\mathrm{e}^{-\alpha t^2}$, where α is a positive constant, and that the spacecraft comes to rest at time $t = T$,

(*c*) find, in terms of U and T only, the distance travelled by the spacecraft in decelerating from speed U to rest.

(6)

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5. A uniform rod AB, of mass m and length $2a$, is free to rotate in a vertical plane about a fixed smooth horizontal axis L. The axis L is perpendicular to the rod and passes through the point P of the rod, where $AP = \frac{2}{3}a$.

(*a*) Find the moment of inertia of the rod about L.

(3)

The rod is held at rest with B vertically above P and is slightly displaced.

(*b*) Find the angular speed of the rod when PB makes an angle θ with the upward vertical.

(4)

(*c*) Find the magnitude of the angular acceleration of the rod when PB makes an angle θ with the upward vertical.

(3)

(*d*) Find, in terms of g and a only, the angular speed of the rod when the force acting on the rod at P is perpendicular to the rod.

(5)

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6. (*a*) Prove, using integration, that the moment of inertia of a uniform circular disc, of mass m and radius a, about an axis through the centre of the disc and perpendicular to the plane of the disc is $\frac{1}{2}ma^2$.

(5)

[*You may assume without proof that the moment of inertia of a uniform hoop of mass m and radius r about an axis through its centre and perpendicular to its plane is* mr^2.]

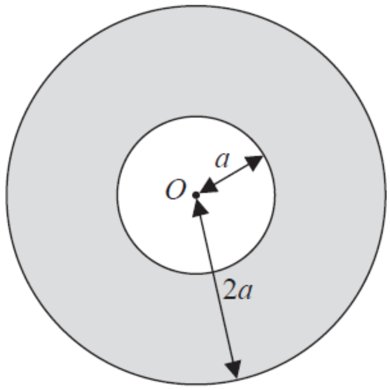

Figure 1

A uniform plane shape S of mass M is formed by removing a uniform circular disc with centre O and radius a from a uniform circular disc with centre O and radius $2a$, as shown in Figure 1. The shape S is free to rotate about a fixed smooth axis L, which passes through O and lies in the plane of the shape.

(*b*) Show that the moment of inertia of S about L is $\frac{5}{4}Ma^2$.

(4)

The shape S is at rest in a horizontal plane and is free to rotate about the axis L. A particle of mass M falls vertically and strikes S at the point A, where $OA = \frac{3}{2}a$ and OA is perpendicular to L. The particle adheres to S at A. Immediately before the particle strikes S the speed of the particle is u.

(*c*) Find, in terms of M and u, the loss in kinetic energy due to the impact.

(8)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\mathbf{F} = \lambda(\pm 8\mathbf{i} \pm 15\mathbf{j})$ $\lambda^2(8^2 + 15^2) = 8.5^2$ $\mathbf{F} = \frac{1}{2}(8\mathbf{i} + 15\mathbf{j})$ $\mathbf{AB} = (5\mathbf{i} + 4\mathbf{j})$ Work done = $\frac{1}{2}(8\mathbf{i} + 15\mathbf{j})$ = 50 (J)	M1 A1 M1 A1 A1 B1 M1 A1 **8**

Notes

First M1 for $\lambda(\pm 8\mathbf{i} \pm 15\mathbf{j})$
First A1 for correct expression
Second M1 for $\lambda^2(8^2 + 15^2)$=8.5^2 from previous incorrect vector
Second A1 for $\lambda = \pm\frac{1}{2}$
Third A1 for correct **F**
B1 for correct **AB**
Third M1 for their **F.AB**
Fourth A1 for 50 (J). (-50 is A0)

Question Number	Scheme	Marks
2.	$\text{IF} = \text{e}^{\int \text{d}t} = \text{e}^t$	M1 A1
	$\dfrac{\text{d}}{\text{d}t}(\mathbf{r}\text{e}^t) = t\text{e}^t\mathbf{i} + \mathbf{j}$	
	$\mathbf{r}\text{e}^t = \int t\text{e}^t\mathbf{i} + \mathbf{j}\ \text{d}t$	M1
	$\mathbf{r}\text{e}^t = (t\text{e}^t - \text{e}^t)\mathbf{i} + t\mathbf{j} + \mathbf{C}$	M1 A2
	$t = 0, \mathbf{r} = \mathbf{i} + \mathbf{j} \Rightarrow \mathbf{C} = 2\mathbf{i} + \mathbf{j}$	M1
	$\mathbf{r}\text{e}^t = (t\text{e}^t - \text{e}^t)\mathbf{i} + t\mathbf{j} + 2\mathbf{i} + \mathbf{j}$	A1
	$\mathbf{r} = (t-1)\mathbf{i} + t\text{e}^{-t}\mathbf{j} + (2\mathbf{i} + \mathbf{j})\text{e}^{-t}$	
	$= (t - 1 + 2\text{e}^{-t})\mathbf{i} + (t+1)\text{e}^{-t}\mathbf{j}$	A1 **9**

Notes

First M1 for IF
First A1 for e^t
Second M1 see scheme
Third M1 for attempt to integrate (must include parts)
A2 for a correct integral
Fourth M1 for use of limits
A1 for a correct **C**
A1 for answer (any equivalent form)

Question Number	Scheme	Marks
3.	$(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) + (\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) + (-3\mathbf{i} + \mathbf{j} + 3\mathbf{k}) = \mathbf{0}$	M1 A1
	$(\mathbf{i} + \mathbf{j} - 2\mathbf{k}) \times (2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) + (3\mathbf{i} - \mathbf{j} - \mathbf{k}) \times (\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) + (\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times (-3\mathbf{i} + \mathbf{j} + 3\mathbf{k})$	M1
	(allow $\sum \mathbf{F} \times \mathbf{r}$)	
	$= (5\mathbf{i} - 3\mathbf{j} + \mathbf{k}) + (-2\mathbf{i} + 5\mathbf{j} - 11\mathbf{k}) + (-7\mathbf{i} - 6\mathbf{j} - 5\mathbf{k})$	A3
	$= (-4\mathbf{i} - 4\mathbf{j} - 15\mathbf{k})$	A1
	$\sqrt{(-4)^2 + (-4)^2 + (-15)^2}$	M1
	$\sqrt{257}$ Nm (2 SF or better)	A1 **9**

Notes

First M1 for $\Sigma \mathbf{F}_i$
First A1 for $= \mathbf{0}$
Second M1 for $\Sigma\ \mathbf{r} \times \mathbf{F}$ or $\Sigma\ \mathbf{F} \times \mathbf{r}$
A3 -1 e.e.o.o. (-1 per cross product)
A1 $\pm(-4\mathbf{i} - 4\mathbf{j} - 15\mathbf{k})$
Third M1 for $|\mathbf{G}| = \sqrt{(-4)^2 + (-4)^2 + (-15)^2}$
A1 for $\sqrt{257}$ or 2sf or better.

Question Number	Scheme	Marks
4. (a)	$(m+\delta m)(v+\delta v)+(-\delta m)(v+k)=mv$ $mv+v\delta m+m\delta v-v\delta m-k\delta m=mv$ $m\delta v-k\delta m=0$ $\frac{\mathrm{d}v}{\mathrm{d}m}-\frac{k}{m}=0$	M1 A2 **DM**1 A1 (5)
(b)	$\int \mathrm{d}v = k\int \frac{\mathrm{d}m}{m}$ $v=k\ \ln m + C$ $v=U, m=M \Rightarrow C=U-k\ln M$ $v=k\ \ln m+U-k\ln M$ $v=U+k\ln\left(\frac{m}{M}\right)$ $v=0\Rightarrow m=Me^{-\frac{U}{k}}$	M1 A1 M1 A1 M1 A1 (6)
(c)	$m=Me^{-\alpha t^2}\Rightarrow v=U-k\alpha t^2$ $s=Ut-\frac{1}{3}k\alpha t^3(+D)$ At $t=T$, $s=UT-\frac{1}{3}k\alpha T^3$ At $t=T, v=0\Rightarrow k\alpha T^2=U$ $s=UT-\frac{1}{3}UT=\frac{2}{3}UT$	M1 M1 A1 M1 M1 A1 (6) **17**

Notes	
4.(a)	First M1 for momentum equation (correct number of terms, excluding any $\delta m \delta v$ terms) A2 for a correct equation -1 e.e. Second M1, dependent on first M1, for simplifying and dividing by $m\delta m$ and taking limits Third A1 for PRINTED ANSWER
4.(b)	First M1 for separating and integrating First A1 correct expression (without C) Second M1 for using limits Second A1 for a correct v (seen or implied) Third M1 for putting $v = 0$ and solving for m Third A1 for correct answer
4(c).	First M1 for obtaining $v = U - k\alpha t^2$ (method) Second M1 for integrating wrt time First A1 for a correct expression for s (without D) Third M1 for using $v = 0$ at $t = T$ to obtain $U = k\alpha T^2$ (method) Fourth M1 for obtaining s in terms of U and T Second A1 for correct answer

Question Number	Scheme	Marks
5.(a)	$I_L = \frac{1}{3}ma^2 + m(\frac{1}{3}a)^2$ $= \frac{4}{9}ma^2$	M1 A1 A1 (3)
(b)	$\frac{1}{2}\frac{4}{9}ma^2\dot{\theta}^2 = mg\frac{1}{3}a(1-\cos\theta)$ $\dot{\theta} = \sqrt{\frac{3g(1-\cos\theta)}{2a}}$	M1 A1 A1 A1 (4)
(c)	$mg\frac{1}{3}a\sin\theta = \frac{4}{9}ma^2\ddot{\theta}$ $\frac{3g\sin\theta}{4a} = \ddot{\theta}$	M1 A1 A1 (3)
(d)	$mg\cos\theta - X = m\frac{1}{3}a\dot{\theta}^2; X = 0$ $\dot{\theta}^2 = \frac{3g(1-\cos\theta)}{2a}$ eliminating $\cos\theta$ and solving, $\dot{\theta} = \sqrt{\frac{g}{a}}$	M1 A1 A1 **DM**1 A1 (5) **15**

Notes

5.(a) M1 for use of parallel axes rule
First A1 for correct expression
Second A1 for answer

5.(b) M1 for energy equation
First A1 for KE terms
Second A1 for PE terms
Third A1 for answer

5.(c) M1 for moments about axis (or differentiate energy equation)
First A1 for a correct equation
Second A1 for answer

5.(d) First M1 for resolving along the rod
First A1 for forces incl. $X = 0$
Second A1 for mass x accln
Second M1, dependent on first M1, for eliminating $\cos\theta$ and solving for $\dot{\theta}$
Third A1 for correct answer

Question Number	Scheme	Marks
6.(a)	$\delta m \approx 2\pi x\delta x \dfrac{m}{\pi a^2} = \dfrac{2mx\delta x}{a^2}$	M1 A1
	$\delta I \approx \dfrac{2mx^3\delta x}{a^2}$	A1
	$I = \dfrac{2m}{a^2}\int_0^a x^3 \, \mathrm{d}x$	M1
(b)	$= \tfrac{1}{2}ma^2$ PRINTED	A1 (5)
	$\tfrac{1}{2}\dfrac{4M}{3}(2a)^2 - \tfrac{1}{2}\dfrac{M}{3}a^2$	M1 A1
	$= \tfrac{5}{2}Ma^2$	
	$2I = \tfrac{5}{2}Ma^2$ (perp axes rule)	M1
	$I = \tfrac{5}{4}Ma^2$ PRINTED	A1 (4)
	$\left(\tfrac{5}{4}Ma^2 + M(\tfrac{3a}{2})^2\right)\omega = Mu(\tfrac{3a}{2})$	M1 A1 A1
(c)	$\omega = \tfrac{3u}{7a}$	A1
	KE loss $= \tfrac{1}{2}Mu^2 - \tfrac{1}{2}\left(\tfrac{5}{4}Ma^2 + M(\tfrac{3a}{2})^2\right)(\tfrac{3u}{7a})^2$	M1 **A2 ft**
	$= \dfrac{5Mu^2}{28}$	A1 (8)
		17

Notes	
6.(a)	First M1 for area element First A1 for a correct δm Second A1 for a correct δI Second M1 for using mass per unit area and integrating with correct limits Third A1 for the **PRINTED ANSWER**
6.(b)	M1 for use of difference of MI (difference in the masses must be *M*) A1 for correct expression without mass per unit area M1 for use of MI about diameter (in formula book) A1 for the **PRINTED ANSWER** **N.B.** The two M marks may be earned in either order
6.(c)	First M1 for conservation of angular momentum equation First A1 for LHS on scheme Second A1 for RHS on scheme Third A1 for a correct ω (or possibly v) Second M1 for a difference in KE (must have found an ω) (omission of MI of particle is missing term so M0) Fourth and fifth A2 **ft** on their ω Sixth A1 for a correct positive answer